# A STUDY OF GAS FLOW OVER A ROUGH SURFACE

M. V. Khanin

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 7, No. 6, pp. 88–93, 1966

The erosion of materials is closely connected with the type of gas or liquid flow over their surfaces. In this paper we shall consider gas flow over a rough surface, and the effect of this on the material, from the point of view of erosion.

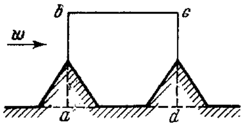


Fig. 1

**1. Fundamentals.** When a gas flows over a solid surface, it may give rise to mechanical disintegration and removal of the material in its initial state, i.e., erosion. To investigate the mechanism of erosive disintegration of materials we must know something about the gas-solid interaction. Studies of the surface geometry of polycrystalline materials such as, for example, graphite, have shown that the surface always becomes rougher as a result of erosion (the only exception occurs for materials on which a continuous liquid film is formed). The properties of gas flows over rough surfaces are important in connection with hydraulic resistance, heat transfer, and certain other problems.

It is known [1] that flow-velocity fluctuations, which have a periodic character in a very broad range of mean flow velocity u, occur at the wall. The velocity fluctuations occur even for very small values of the Reynolds number R, i.e., in the laminar region. The dependence of the fluctuation frequency $\omega$ on u at low velocities, i.e., small Reynolds numbers, can be represented by a straight line to a good approximation [1]. Since the experiments leading to this result were carried out in the same tube, it follows that the Strouhal number S can be regarded as constant for small Reynolds number R. Velikanov [1] found that S = 0.45. Experimental evidence [1] leads to the conclusion that the flow fluctuations are due to eddies which appear periodically at the wall and spread from it.

Rough surfaces of many polycrystalline materials used in practice have irregularities whose height is of the order of a few hundredths of a millimeter, so that direct studies of gas flow within such irregularities are obviously impossible. The only alternative is to model the gas flow over a rough surface.

**2. Model conditions.** It is well known that, in the immediate neighborhood of a solid wall, the velocity profiles in the boundary layer can be regarded as rectilinear regardless of the nature of the flow (laminar or turbulent), and it is always possible to find a region near the wall where the Mach numbers are small and changes in density and temperature can be neglected. Boundary-layer calculations show that the linear dimensions of such regions are of the order of the height of surface irregularities. The flow of a viscous gas in the immediate neighborhood of a solid surface can therefore be approximately regarded as the flow of an incompressible fluid, i.e., viscous flow described by the Navier-Stokes equations without convective terms.

Studies of gas flow over a rough surface can be carried out on the assumption that the incident flow is stationary. The undisturbed flow is defined as a flow whose parameters can be determined by calculation without allowance for the effect of surface roughness. Stationarity of these parameters is taken to mean that the mean values of the actual flow parameters, which are equal to the calculated values, are time-independent.

It is well known that the velocity and pressure fields in a flow are determined by the boundary velocity distribution [2]. One need then specify only the velocity profile on the entrance section of the boundary of the region under investigation, where it can be controlled.

Therefore, the space boundary $abcda$ can be divided into the following characteristic regions (Fig. 1): $ab$ (or even $abc$)—entrance region where the velocity profile can be controlled; cd—exit region where the velocity profile is the same or nearly the same as the velocity profile on $ab$, and $da$—region coinciding with the surface of the solid wall where the adhesion conditions are satisfied.

Under these conditions the equations of motion and continuity, and the boundary conditions, can be written in the form

$$\rho \partial \mathbf{w}/\partial t = -\operatorname{grad} p + \mu \nabla^2 \mathbf{w},$$
$$div\ \mathbf{w} = 0,\ \mathbf{w}_{ab} = \psi\ (\mathbf{l}_{ab},\ w^*), \tag{2.1}$$

where $\rho$ is the density, p the excess pressure, w the velocity vector, t the time, $\mu$ the dynamic viscosity, w* the parametric value of the velocity, $l$ the radius vector, and $\nabla^2$ the Laplace operator.

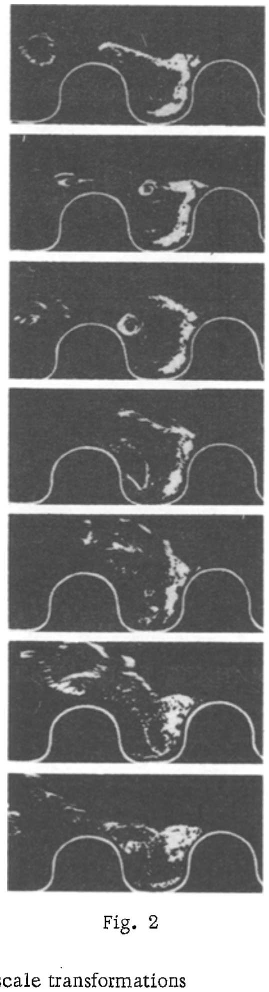

Fig. 2

We shall use the scale transformations

$$\mathbf{l} = l_0\mathbf{L},\quad \mathbf{l}_{ab} = l_0\mathbf{L}_{ab},\quad t = t_0T,\quad \mathbf{w} = w_0\mathbf{W},$$
$$w^* = w_0W^*,\quad \mu = \mu_0\mathrm{M},\ p = p_0P,\ \rho = \rho_0R,$$

so that the above equations and boundary conditions can be written in the dimensionless form

$$\left(\frac{\rho_0 w_0}{t_0}\right) R \left(\frac{\partial \mathbf{W}}{\partial T}\right) = -\left(\frac{p_0}{l_0}\right) \operatorname{grad} P + \frac{\mu_0 w_0}{l_0^2} \mathrm{M}\nabla^2\mathbf{W},$$
$$\left(\frac{w_0}{l_0}\right) \operatorname{div} \mathbf{W} = 0,\qquad w_0\mathbf{W}_{ab} = \psi\ (l_0\mathbf{L}_{ab},\ w_0W^*), \tag{2.2}$$

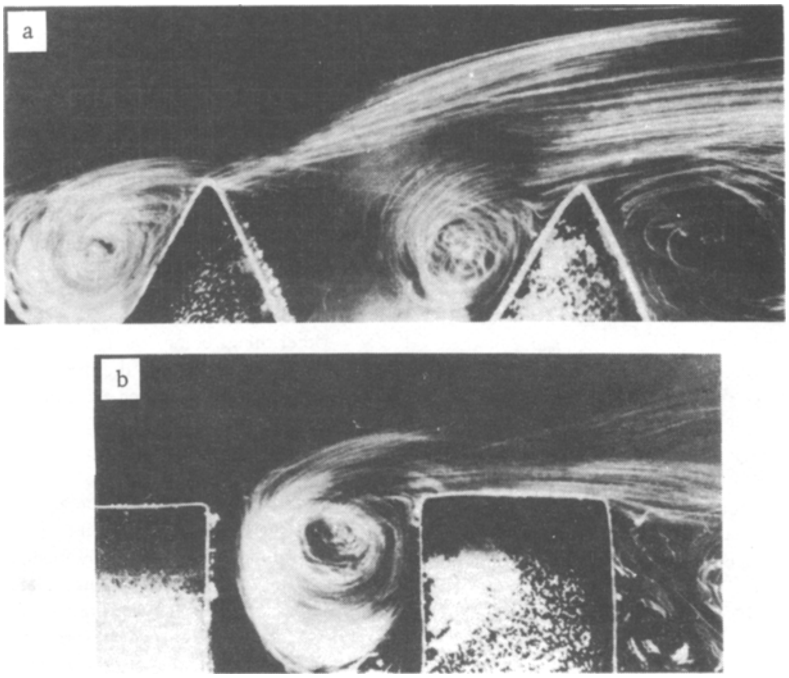


Fig. 3

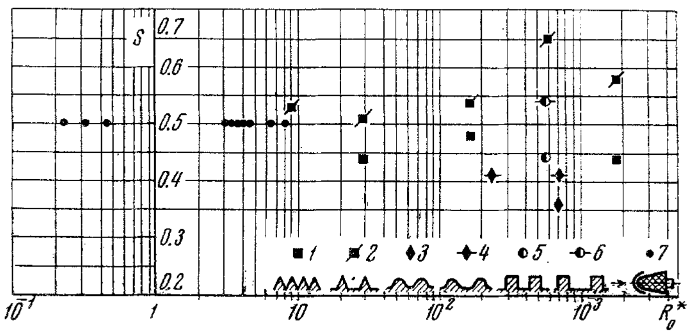


Fig. 4

Hence we have

$$\rho_0 w_0/t_0 = p_0/l_0 = \mu_0 w_0/l_0^2. \qquad (2.3)$$

Since among the dimensional physical quantities in the differential equations and boundary conditions, only three have independent dimensions, it follows that one further relation must be found to determine the remaining quantities in terms of these three. This third relation can be taken to be

$$p_0 = \rho_0 w_0^2. \qquad (2.4)$$

The scales with independent dimensions will be taken to be: $l_0$, the mean height of surface irregularities, and $w^*$, the velocity of the undisturbed flow at this height.

We then have $W^* = 1$ and $M = 1$. The remaining scales and dimensionless quantities will then be expressed in terms of the above arbitrary quantities as follows:

$$p_0 = \frac{\mu w^*}{l_0}, \quad \rho_0 = \frac{\mu_0}{l_0 w^*}, \quad t_0 = \frac{l_0}{w^*}, \quad \mathbf{L} = \frac{\mathbf{l}}{l_0},$$

$$\mathbf{L}_{ab} = \frac{\mathbf{l}_{ab}}{l_0}, \quad W = \frac{\mathbf{w}}{w^*}, \quad R = \frac{\rho l_0 w^*}{\mu} = R_0^*,$$

$$T = \frac{t w^*}{l_0} = \frac{1}{S_0^*}, \quad P = \frac{p l_0}{\mu w^*} = E R_0^*.$$

It is known that the time in the dimensionless complex $S_0^*$ represents the period of the oscillations. We shall therefore interpret t as the period of fluctuations which can occur during the flow of a gas (liquid) over a rough surface.

The dimensionless form of the differential equations and boundary conditions is then

$$R_0^* (\partial \mathbf{W} / \partial T) = -\operatorname{grad} P + \nabla^2 \mathbf{W}, \quad \operatorname{div} \mathbf{W} = 0,$$
$$\mathbf{W}_{ab} = \psi(\mathbf{L}_{ab}). \qquad (2.5)$$

The solution of these differential equations satisfying the boundary conditions can be written in the form

$$T = F_T(\mathbf{W}, \mathbf{L}, \mathbf{W}_{ab}, R_0^*), \qquad P = F_P(\mathbf{W}, \mathbf{L}, \mathbf{W}_{ab}, R_0^*), \quad (2.6)$$

but $\mathbf{W} = \psi(\mathbf{W}_{ab}, \mathbf{L})$, and therefore,

$$T = F_T(\mathbf{W}_{ab}, \mathbf{L}, R_0^*), \qquad P = F_P(\mathbf{W}_{ab}, \mathbf{L}, R_0^*), \quad (2.7)$$

or, written out in full,

$$\frac{1}{S} = F_T\left(\frac{\mathbf{w}_{ab}}{w^*}, \frac{\mathbf{l}}{l_0}, R_0^*\right),$$
$$E R_0^* = F_P\left(\frac{\mathbf{w}_{ab}}{w^*}, \frac{\mathbf{l}}{l_0}, R_0^*\right). \qquad (2.8)$$

Hence, it is clear that similarity can be achieved by ensuring the same distribution of $\mathbf{W}_{ab}$, L, and $R_0^*$ in the model and in the specimen. This will ensure that the T and P criteria will be identical, i.e.,

$$T_1 = T_2 \quad \text{or} \quad (S_0^*)_1 = (S_0^*)_2, \qquad (2.9)$$

$$P_1 = P_2 \quad \text{or} \quad (E R_0^*)_1 = (E R_0^*)_2, \qquad (2.10)$$

where subscripts 1 and 2 represent the model and the specimen, respectively.

From Eq. (2.9) we can obtain the relation between the fluctuation frequency $\omega = 1/t$ in the model and in the specimen

$$\omega_1 = \omega_2 \frac{(w^*/l_0)_1}{(w^*/l_0)_2}. \qquad (2.11)$$

Since the velocity profile for the undisturbed flow over a rough surface can be assumed to be rectilinear, it follows that when the velocity profile in the model is also rectilinear, we have identical boundary velocity distributions on the entrance section of the boundary i.e., $(\mathbf{W}_{ab})_1 = (\mathbf{W}_{ab})_2$.

This means that, in the solution given by Eq. (2.7), we can omit the dimensionless boundary velocity and, consequently,

$$T = F_T(\mathbf{L}, R_0^*), \quad P = F_P(\mathbf{L}, R_0^*). \qquad (2.12)$$

For a rectilinear velocity profile $w^*/l_0 = (dw/dy)_{y=0}$ expression (2.11) becomes

$$\omega_1 = \omega_2 \frac{(dw/dy)_{y=0\,1}}{(dw/dy)_{y=0\,2}}, \qquad (2.13)$$

and the frequency of the flow fluctuations can be determined from

$$\omega = S_0^* \, (dw/dy)_{y=0}. \qquad (2.14)$$

It is important to note that, as far as simulation of gas flow over rough surfaces is concerned, the use of differential equations, and of their solution satisfying the boundary conditions given by (2.12), is valid for different viscous flow conditions. However, each of these flows occurs because viscous flow is self-similar in a definite range of values of $R_0^*$. Therefore, the numbers $R_0^*$ for creep will differ from the values of $R_0^*$ for which we have laminar flow in a tube, or flow in the laminar sublayer of the boundary layer. Consequently, the flow of a liquid or gas in a model will be similar to the flow in the specimen only if the values of $R_0^*$ are in the range in which a given viscous flow takes place.

Other things being equal, the simulation possibilities are restricted by the minimum size of the surface irregularities, which is determined by the height below which the equations for a continuous medium must be treated with caution, or cannot be used at all, since the mean free path of the molecules may be comparable with the linear dimensions of the surface irregularities.

**3. Model experiments.** The gas flow over a rough surface was simulated using the relations given by (2.12). The experiments were carried out in a water tank, using models which were geometrically similar to the surface profile of irregularities on the rough surface of graphite models which were taken as the specimens. Therefore, $\mathbf{L}_1 = \mathbf{L}_2$. The roughness models were set up near the side wall of the tank in such a way that the current of water incident on them was steady. The edges of the model were just above the water level.

The undisturbed flow had a rectilinear velocity profile at the wall (this profile was determined in preliminary experiments). This was used to ensure that the boundary-velocity distribution was the same in the model and in the specimen, i.e., $(W_{ab})_1 = (W_{ab})_2$. The size of the models and the velocity of the undisturbed flow were chosen so that $(R_0^*)_1 = (R_0^*)_2$.

We have also carried out experiments with $(R_0^*)_1$ greater or less than $(R_0^*)_2$ for the same boundary-velocity distribution of the undisturbed flow. In these experiments we varied only the height of the model, retaining the geometric similarity. Moreover, we have carried out experiments for constant linear dimensions of the models but different absolute magnitudes of the velocity gradients in the undisturbed flow at the wall. The numbers $(R_0^*)_1$ were varied in the range $1 < (R_0^*)_1 < 2000$, and the height $l_0$ of the models in the range between 4 and 32 mm.

Long-exposure and motion-picture photography of the water flow around the models was carried out. Aluminum powder was used to make the state of the water surface visible.

The motion of the liquid around the roughness model was found to be analogous to the gas (liquid) flow in separation zones [3-6].

The essence of this can be summarized as follows: a separation boundary is initially established between the external flow and the liquid in the troughs of the model, and this eventually becomes unstable due to the eddies which break off from the front wall, of the trough, decay, and give rise to a turbulent separation layer [7].

Experiments have shown that the liquid in the trough is brought into rotational motion, and the external eddies moving along the separation layer periodically penetrate the trough. This is associated with an increase in the amount of liquid in the trough, and a time is eventually reached when the equilibrium between the liquid and the incident flow is violated, the separation layer breaks off from the rear (relative to the flow) wall of the trough, and the excess mass is ejected into the incident flow. Equilibrium is then re-established, the separation layer closes again on the rear wall of the trough, and the entire cycle is repeated once again.

We have thus established that the flow of the liquid in the troughs of the roughness model, and hence the flow of a gas in the troughs of a rough surface, has a fluctuational character.

Figure 2 illustrates the motion of the liquid in one of the models, during the fluctuation period, while Figs. 3a and 3b show flow photographs for the two models investigated in water-tank experiments.

Analysis of the experiments has resulted in numerical values for the fluctuation frequency in the troughs of the model and the Strouhal numer. Figure 4 shows the results of water-tank tests on the roughness models whose profiles are indicated, and on the graphite models in high-temperature flows of argon and air. The notation is as follows:

| Points | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\omega$ = | 0.9 | 0.4—1.1 | 0.7 | 0.3—0.8 | 0.9 | 1.1 | (0.6—3.8) $10^6\,\mathrm{sec}^{-1}$. |

**4. Analysis of results.** It is clear that velocity fluctuations in the troughs are accompanied by a change in the parameters of the liquid (gas) and, in particular, there is a change in the pressure along the walls [5, 6]. The net result of this is a periodic variation in the pressure over the crests of the model; this appears to be responsible for erosion of the surface.

It is clear from Fig. 4 that the Strouhal numbers depend on the shape and the relative size of the surface irregularities, but do not depend on the number $R_0^*$ for a given roughness, within the limits investigated in the water-tank experiments. Consequently, for a given boundary-velocity distribution in geometrically similar models, the fluctuation frequency should be independent of their size. This is confirmed by experiments carried out on models of different size with constant boundary distribution of the undisturbed flow velocity. Experiments with different velocity gradients in the undisturbed flow show that the fluctuation frequency is directly proportional to the velocity gradient, confirming the results given by Eq. (2.14).

The experimentally established similarity between flows at different Reynolds numbers confirms that the self-similarity conditions of Eq. (2.12) can therefore be written in an even simpler form, and namely:

$$T = F_T(\mathrm{L}),\; P = F_P\;(\mathrm{L}), \tag{4.1}$$

and the similarity of such flows will occur only when the rough surfaces are geometrically similar, the boundary velocity distribution is rectillinear, and the numbers $R_0^*$ are such that the given viscous flow takes place.

We note that the dependence of the fluctuation frequency in the surface troughs on the velocity gradient in the undistrubed flow occurs not only in the case of an external flow over a rough surface, for example, in fluids flowing past conical graphite models, but also for flows over the inner surface of a tube. In fact, for gases flowing in a tube we have

$$(dw\,/\,dy)_{y=0} = {}^1/_8\; \rho u^2\lambda/\mu, \tag{4.2}$$

which is valid for all degrees of roughness of its walls [8] ($\lambda$ represents frictional losses).

For a laminar flow in a tube of diameter d, and $R = ud\rho/\mu$, we have

$$\lambda = 64\; R^{-1}. \tag{4.3}$$

If we substitute for $\lambda$ and R in Eq. (4.2) we obtain

$$(dw/dy)_{y=0} = 8\; u/d$$

and

$$\omega = 8\; S_0{}^* u/d. \tag{4.4}$$

It is thus clear that the fluctuation frequency in the surface troughs in the tube wall can be determined either from the velocity gradient of the undisturbed flow at the wall, or from u. Let us verify this using the data given in [1] which show that, for small R,

$$\omega = 0.6\; u. \tag{4.5}$$

We must now find the dependence of $\omega$ on u, using Eq. (4.4). The Strouhal number in this expression is

$$S_0{}^* = \omega l_0 / w^*. \tag{4.6}$$

Let us express the velocity $w^*$ and the height $l_0$ of the surface crests in terms of u and the tube radius r. The velocity profile in the tube in the case of laminar flow is described by the Poiseuille parabola

$$w = w_0\,(1 - a^2/r^2), \tag{4.7}$$

where $w_0 = 2u$ is the maximum flow velocity [8] and $a$ is the modulus of the position vector of points on a circle in the cross-sectional plane in which the flow velocity is equal to w.

Hence,

$$w^* = 2u\;\;[1 - (r - l_0)^2/r^2], \tag{4.8}$$

or, neglecting $\lambda_0^2/r^2$ in comparison with $2l/r$, we have

$$w^* = 2u\,(2l_0/r - l_0{}^2/r^2) \approx (4l_0/r)u. \tag{4.9}$$

Substituting Eq. (4.9) into (4.6), we obtain the relation between $S_0^*$, which characterizes the pulsation frequency in the troughs on the rough surface, and the Strouhal number S:

$$S_0{}^* = {}^1/_4 S. \tag{4.10}$$

Using Eq. (4.10), let us transform Eq. (4.4) into the following relation between $\omega$ and u:

$$\omega = 2\;\; Su/d. \tag{4.11}$$

In the experiments described in [1], S = 0.45 and d = 1.5 cm. Consequently,

$$\omega = 0.6u,$$

which is identical with Eq. (4.5).

From the experimental results shown in Fig. 4 it follows that, the fluctuation frequency near irregularities with edges that are less sharp is characterized by smaller Strouhal numbers $S_0^*$. In the experiments described in [1], $S_0^* = 0.11$, which indicates that the surface of the ebonite tube used in these experiments was sufficiently smooth.

Since roughness models tested in the water tank are similar to the irregularities on the surface of graphite models, we may consider that the most probable values of the Strouhal number for these irregularities will like in the range used in water-tank experiments, i. e., $S_0^* = 0.45$–$0.55$, and the fluctuation frequency can be determined from Eq. (2.14). The gas-flow fluctuation frequency determined in the experiments with graphite models, which were carried out on the assumption that the frequency of these fluctuations was equal to the proper oscillations of the individual grains, resulted in the value $S_0^* = 0.5$ (Fig. 4).

The author is grateful to G. I. Petrov for his interest in this work.

REFERENCES

1. M. A. Velikanov, Channel-flow Dynamics [in Russian], Gidrometeoizdat, 1946.

2. L. G. Eigenson, Simulation [in Russian], Izd. Nauka, 1952.

3. M. G. Morozov, V. M. Eroshenko, and Yu. N. Petrov, "Flow in stagnation zones on the surface of bodies exposed to ultrasonic flow of air," collection: Physical Gasdynamics and Heat Transfer [in Russian], Izd-vo AN SSSR, 1961.

4. S. S. Kutateladze, et al., Heat Transfer and Friction in a Turbulent Boundary Layer [in Russian], Izd. SO AN SSSR, 1964.

5. M. Kawaguti, "Numerical solution of the Navier-Stokes equation of the flow in two dimensional cavity," J. Phys. Japan, vol. 16, no. 11, 1961.

6. A. F. Charwat, C. F. Dewey Jr., Y. N. Roos, and Y. A. Hitz, "An investigation of separated flows," J. Aerospace Sci., vol. 28, no. 6, 7, 1961.

7. L. Prandtl, Gasdynamics [Russian translation], Izd-vo inostran. lit., 1949.

8. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Fizmatgiz, 1959.

26 November 1965 Moscow